

## MM－KBAC：Using mixed models to adjust for population structure in a rare－variant burden test

Tuesday，June 10， 2014

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## Golden Helix Offerings



## Services

- Genomic Analytics
- Genotype Imputation
- Workflow

Automation

- SVS Certification \& Training



## Software

- SNP \& Variation Suite (SVS) for NGS, SNP, \& CNV data
- GenomeBrowse
- New products in development



## Support

- Support comes standard with software
- Customers rave about our support
- Extensive online materials including tutorials and more


## SNP \& Variation Suite (SVS)



## Core Features

- Powerful Data Management
- Rich Visualizations
- Robust Statistics
- Flexible
- Easy-to-use


## Applications

- Genotype Analysis
- DNA sequence analysis
- CNV Analysis
- RNA-seq differential expression
- Family Based Association


## Timeline



## Study Design

- Large cohort population based design (cases with matched controls or quantitative phenotypes and complex traits)
- Assumes: independent and well matched samples
- Can interrogate complex traits
- Small families (trios, quads, small extended pedigrees)
- Can only analyze a single family at a time, looking for de Novo, recessive or compound het variants unique to an affected sample in a single family
- Looking for highly penetrant variants


## What If????

- What if we have:
- Known population structure
- Cannot guarantee independence between samples
- Controls were borrowed from a different study
- Multiple families with affected offspring all exhibiting the same phenotype
- Multiple large extended pedigrees of unknown structure


## Just Add Random Effects!

- Why can't we just add random effects to our regression models for our rare-variant burden testing algorithms?
- Existing mixed model algorithms assume a linear model
- Kernel-based adaptive clustering (KBAC) uses a logistic regression model
- Hmm what to do....?


## WARNING!

What is about to follow are formulas and statistics, specifically matrix algebra...

But don't worry we'll end the webcast with a presentation of some preliminary results! So hang in there!

## But first....

The dataset we have chosen for today is the 1000 Genomes Pilot 3 Exons dataset with a simulated phenotype.



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## Relatedness of samples

Population == 'CEU'Population == 'CHB'Population == 'CHD'Population $==$ 'JPT'Population == 'LWK'
Population == 'TSI'
| Population == 'YRI'


## Why Mixed Models + KBAC?

- OK Mixed Models makes sense, but why KBAC?
- KBAC was chosen as our proof of concept rare-variant burden test for complex traits
- KBAC uses a score test which is trivial to calculate once you compute the reduced model
- Mixed models can be added to other burden and kernel tests using the same principles


## What is KBAC?

- KBAC = Kernel-based Adaptive Clustering
- Catalogs and counts multi-marker genotypes based on variant data
- Assumes the data has been filtered to only rare variants
- Performs a special case/control test based on the counts of variants per region (aka gene)
- Test is weighted based on how often each genotype is expected to occur according to the null hypothesis
- Genotypes with higher sample risks are given higher weights
- One-sided test primarily, which means it detects higher sample risks

A Novel Adaptive Method for the Analysis of NextGeneration Sequencing Data to Detect Complex Trait Associations with Rare Variants Due to Gene Main Effects and Interactions

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Introduction
Currently there is great interest in investigating the etiology of complex disease due to rare variants [1-6]. Until recently, indirec mapping of common vaniants has been the emphasis of complex
trait association studies. It has been demonstrated that common variants tend to have modest phenotypic effects while rare variants are likely to have stronger phenotypic effects [7], although not trong enough to cause familial aggregation [3]. For mapping complex diseases duc to conmmon variants, instead of genotypin
functional variants, tagsNPs are genotyped which act as a prox functional variants, tagsNPs are genotyped which act as a proxy
for the underlying causal variants. For rare variant association studies, indirect mapping is not an optimal approach due to low correlations $\left(r^{2}\right)$ between tagSNPs and rare variants. Instead, direct mapping should be used, where functional variants are analyzod
In order to implement direct mapping, variants must first be identified. Large scale sequencing efforts have begum induding the 1000 Genome Project, which will provide a better undestanding
of the allelic architecture of the genome and a detailed catalog of
human variants. Next-generation sequencing technologies e.g.
Roche 454, ABI SOI 1 D , and Illumina HiSco havr made frasible to carry-out rare variant association studies of candidate regions, exomes and genomes. Gene interactions are believed to be involved in a broad spectrun of complex discase etiologies [9]. Although a number of metho
have been developed to detect gene interactions betwer variants [ $10-13$ ], their detection has been limited [10]. There evidence that rare variant interaction also plays a role in discas genetic loci are commonly jointly analyzed in order to aggrega genetic loci are commonly jontly analyzed in order to aggr eg
information, for example genes with simila fiunctions or residing in the same pathway [3,4]. Therefore it is necessary to account for potential interactions bectween rare variants in different loci [14] and
interactions between conmon and rare variants
$[15,16]$. Idealb, wen carring out direot maping olv 15 should be testedfor associations. When DNA smmples are sequenced both causal and non-causal vanaants are uncovered. Bioinformatio

## Pictorial Overview of Theory


$\begin{array}{ll}\text { Known SNPS } & \mathrm{A} \\ & \mathrm{G}\end{array}$


## C

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## Filter Common/Known SNPs



## Filter by Gene Membership



## Rare Sequence Variants



Known Genes $1 \quad$ —_
Known SNPS
A
G


C
T

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## KBAC Statistic

- $K B A C_{1}=\sum_{i=1}^{k}\left(\frac{N_{i}^{A}}{N^{A}}-\frac{N_{i}^{U}}{N^{U}}\right) K_{i}^{0}\left(\hat{R}_{i}\right)$
- Where the weight is defined as:

$$
w_{i}=K_{i}^{0}\left(\hat{R}_{i}\right)=\int_{0}^{\hat{R}_{i}} k_{i}^{0}(r) d r
$$

The weight can be calculated as a:

- Hyper-geometric kernel
- Marginal binomial kernel
- Asymptotic normal kernel


## Determining the KBAC p-value

- Monte-Carlo Method is used as an approximation for finding the pvalue
- The number of cases $n_{i}^{A}$ for each genotype $G_{i}$ approximates a binomial distribution $n_{i}^{A} \sim \operatorname{Binom}\left(n_{i}, \frac{n^{A}}{n}\right)$
- The case status is permuted among all samples. The covariates and genotypes are held fixed.


## Logistic Mixed Model Equation

$$
\log \left(\frac{P\left(Y_{j}=1 \mid X_{j}, X_{f j l}, u_{j}\right)}{1-P\left(Y_{j}=1 \mid X_{j}, X_{f j l}, u_{j}\right)}\right)=\beta_{0}+\beta_{1} X_{j}+\sum_{l}\left(\beta_{f l} X_{f j l}\right)+u_{j}
$$

Null hypothesis: $H_{0}: \beta_{1}=0$
The score statistic to test the null of the independence of the model from $X_{j}$ is:
$U=\sum_{j} X_{j}\left(Y_{j}-\mu_{j}\right)$, where
$\mu_{j}=h\left(\eta_{j}\right)=\frac{e^{\eta_{j}}}{1+e^{\eta_{j}}}$, and
$\eta_{j}=\beta_{0}+\sum_{l} \beta_{f l} X_{f j l}+u_{j}$, and
$u_{j}$ is the random effect for the $j^{\wedge}(t h)$ sample.

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## Logistic (Reduced) Mixed Model Equation

$$
\log \left(\frac{P\left(Y_{j}=1 \mid X_{f j l}, u_{j}\right)}{1-P\left(Y_{j}=1 \mid X_{f j l}, u_{j}\right)}\right)=\beta_{0}+\sum_{l} \beta_{f l} X_{f j l}+u_{j}
$$

Which can be rewritten as:

$$
E[Y \mid u]=h\left(X_{f} \beta+u\right)=h(\eta)=\mu
$$

And

$$
\operatorname{Var}[Y \mid u]=A=A^{1 / 2} A^{1 / 2}
$$

Where $A$ is the variance of the binomial distribution itself, where

$$
\left\{\begin{array}{l}
A_{j j}=\mu_{j}\left(1-\mu_{j}\right) \text { for } j=1 . . n \\
A_{i j}=0 \text { for } i \neq j
\end{array}\right.
$$

And the linear predictor for the model is $\eta=X \beta+u$ While $h(■)$ is the inverse link function for the model

## Solving the Logistic Mixed Model

Iterate between creating a linear pseudo-model and solving for the pseudo-model's coefficients

$$
h(\eta) \doteq h(\tilde{\eta})+\widetilde{\Delta} X(\beta-\tilde{\beta})+\widetilde{\Delta}(u-\tilde{u})
$$

Where

$$
\widetilde{\Delta}=\left(\frac{\partial h(\eta)}{\partial \eta}\right)_{\widetilde{\beta}, \widetilde{u}}
$$

Rearranging yields

$$
\widetilde{\Delta}^{-1}(\mu-h(\tilde{\eta}))+X \tilde{\beta}+\tilde{u} \doteq X \beta+u
$$

The left side is the expected value, conditional on $u$, of

$$
P \equiv \widetilde{\Delta}^{-1}(Y-h(\tilde{\eta}))+X \tilde{\beta}+\tilde{u}
$$

The variance of $P$ given $u$ is

$$
\operatorname{Var}[P \mid u]=\widetilde{\Delta}^{-1} \tilde{A}^{1 / 2} \tilde{A}^{1 / 2} \widetilde{\Delta}^{-1}
$$

Where

$$
\tilde{A}_{j j}=\tilde{\mu}_{j}\left(1-\tilde{\mu}_{j}\right), \tilde{A}_{i j}=0(\text { for } i \neq j)
$$

## Transform Pseudo-Model to use EMMA

Pseudo-model: $P=X \beta+u+\epsilon$ and $\operatorname{Var}[\epsilon]=\operatorname{Var}[P \mid u]$
NOTE: As an alternative, rather than using the prediction of $u$ from the pseudo-model, we can use the expected value of $u$, which is zero

Want to solve using EMMA (Kang 2008)
Find $T$ such that $\operatorname{Var}[T \epsilon]=I$
So that we can write

$$
T P=T X \beta+T u+T \epsilon
$$

And use EMMA to solve the mixed model

$$
T P=T X \beta+T u+\epsilon^{*}
$$

Where the variance of $\epsilon^{*}$ is proportional to $I$
It can be shown that this is solved by letting

$$
T=\widetilde{A}^{-1 / 2} \widetilde{\Delta}
$$

## Summary of the Algorithm

First pick starting values of $\tilde{\beta}$ and $\tilde{u}$, such as all zeros. Repeat the following steps until the changes in $\tilde{\beta}$ and $\tilde{u}$ are sufficiently small:

1. Find $\tilde{\eta}$ and $\tilde{\mu}$ from the original linear predictor equation and the definition of $h(■)$
2. Find the (diagonal) $\widetilde{\Delta}$ matrix
3. Find the pseudo-model $P$
4. Find the (diagonal) matrix $T$
5. Solve the following for new values of $\tilde{\beta}$ and $\tilde{u}$ using EMMA:

$$
T P=T X \beta+T u
$$

NOTE: The alternative method modifies Step 5 to use EMMA to determine the variance components and to find a new value for $\tilde{\beta}$, while leaving the value of $\tilde{u}$ at its expected value of zero.

After convergence, the alternative method predicts the values of $u$, and computes the final values of $\eta$ and $\mu$ from this prediction

## Computing the Kinship Matrix



## KBAC and MM-KBAC SVS Interface



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## Applying MMKBAC to a real study



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## KBAC vs MM-KBAC QQ Plots



## MM-KBAC



KBAC w Pop. Covariates: $\tilde{\lambda}=0.902$

## Signal at PSRC1



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## Signal at HIRA

COSMIC Genes v67, COSMIC
CMIM Genes 2010-10-27, UCSC
KBAC

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## Conclusion

- This will method will be added into SVS in the near future...

In the meantime...

- Like to try it out on your dataset - ask us to be part of our earlyaccess program!
- We have submitted an abstract to ASHG, hope to see you there!


## Announcements

- Webcast recording and slides will be up on our website tomorrow.
- T-shirt Design Contest! Details at www.goldenhelix.com/events/tshirtcontest.html
- Next scheduled webcast is July $\mathbf{2 2 n d}^{\text {nd }}$, but Heather Huson of Cornell University.


## Questions?

## Use the Questions pane in your GoToWebinar window



